8.6

Counting Principles

Slips of paper with the numbers 1-20 inclusive are placed into a hat.

How many ways can you pick an odd number?

10 ways

A prime number?

9 ways

A prime or an odd number?

11 ways

A prime # and odd number?

8 ways

Slips of paper with the numbers 1-20 inclusive are placed into a hat.

An integer divisible by 5?

4 ways

Two integers whose sum is 8?

1 7
2 6
3 5
4 4 replace?

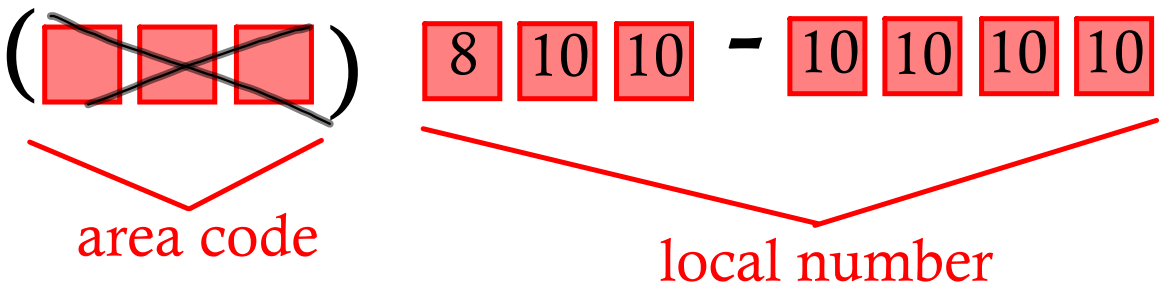
Fundamental Counting Principle

Let E_1 and E_2 be two events. The first event can occurin m_1 different ways. After the first event occurredhe second event can occur in m_2 different ways. Thenumber of ways the two events can occur is $m_1 \times m_2$.

$7 \cdot 4 \quad 28$

EXAMPLE: Fundamental Counting Principle

Telephone numbers in the United States currently have 10 digits. The first three numbers are the area code and the next seven are the local telephone number. How many different telephone numbers are possible within each area code? (note: that at this time, a local telephone number cannot begin with 0 or 1.)



$$8 * 10 * 10 * 10 * 10 * 10 * 10 = 8{,}000{,}000$$

EXAMPLE: In 2000, Missouri adopted new license plates:

The state of Missouri had license plates that consisted of three numbers and then three letters. How many different license plates could the state have?

10 10 10 26 26 26

17,576,000

EXAMPLE: The English alphabet has 26 different letters

How many different pairs of letters from the English alphabet are possible?

a) if you can repeat letters

$26 \cdot 26 =$

b) if you cannot repeat letters

$26 \cdot 25 =$

EXAMPLE: Kentucky Derby Winners

20 horses compete annually in the Kentucky Derby. How many ways can the horses place 1st, 2nd, and 3rd?

$\underset{1st}{20} \quad \underset{2nd}{19} \quad \underset{3rd}{18} = 6,840$

*order matters!

Permutations

A permutation of n things is an ordering of the elements such that **ORDER MATTERS**, and there is a first, second, third, and so on.

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$$_{n}P_{r} = \frac{n!}{(n-r)!}$$

total → n ; r ↑ how many you're choosing

*But we can continue to use the slot method to help us out. Permutations and the fundamental counting principle are alike.

*Sometimes the formula is not always beneficial. Sometimes a problem, where order matters fits itself to the Fundamental Counting Rule and not Permutation

EXAMPLE: Kentucky Derby Winners

20 horses compete annually in the Kentucky Derby. How many ways can the horses place 1st, 2nd, and 3rd?

1st	2nd	3rd
20	19	18

$20 * 19 * 18 = 6840$

$$_{20}P_3 = \frac{20!}{17!} = \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdots}{17}$$

EXAMPLE: Standing in line

There are five people waiting in line for one bathroom. In how many different ways can they stand in line?

5 4 3 2 1

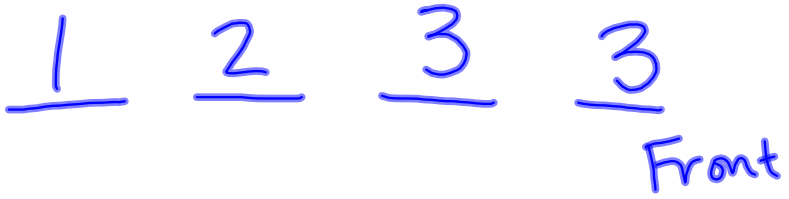
120 ways

$${}_5P_5 = \frac{5!}{5-5!} = \frac{5!}{0!} = \frac{5!}{1}$$

$0! = 1$

EXAMPLE: The Log Flume

You're at Six Flags with three friends. You're all going to ride the Log Flume at the same time, but you're the only one that won't sit in front. In how many ways can you all sit in the log?



Distinguishable Permutations

Suppose a set of n objects has n_1 of one kind of objects, n_2 of a second kind, n_3 of a third and so on.

The number of distinguishable permutations is:

$$\frac{n!}{n_1!\bullet n_2!\bullet n_3!\bullet\ldots}$$

EXAMPLE: Rewriting Banana

In how many distinguishable ways can the letters in the word BANANA be written?

BANANA: six letters, three A's, two N's, and one B

$$\frac{6!}{3!\bullet 2!\bullet 1!} = \frac{6\bullet 5\bullet 4\bullet \cancel{3!}}{\cancel{3!}\bullet 2!} = 60$$

EXAMPLE: Rewriting Google

In how many distinguishable ways can the letters in the word "Google" be written?

$$\frac{6!}{2!\,2!} = \frac{6 \cdot 5 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{2 \cdot 1} \cdot \cancel{2 \cdot 1}}$$

$$= 30$$

EXAMPLE: Rewriting Basketball

In how many distinguishable ways can the letters in the word "basketball" be written?

$$\frac{10!}{2!\,2!\,2!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2}{2\cdot 2\cdot 2}$$

$=$

EXAMPLE: Combinations of n elements, taken r at a time

In how many different ways can three letters be chosen from the letters A, B, C, D, and E?
(the order of the three letters is not important)

ABC
ABD
ABE
ACD
ACE
ADE
BCD
BCE
BDE
⋮

10 ways

Combinations

The number of combinations of n things taken r at a time is given by:

$$ {}_nC_r = \frac{n!}{(n-r)!r!} $$

total

how many you're choosing

*Order does not matter!!

EXAMPLE: Combinations of n elements, taken r at a time

In how many different ways can three letters be chosen from the letters A, B, C, D, and E?
(the order of the three letters is not important)

5 elements, taken 3 at a time

$$_5C_3 = \frac{5!}{2!3!} = \frac{5 \quad 4 \quad \cancel{3!}}{2 \quad 1 \quad \cancel{3!}} = \frac{20}{2} = 10$$

EXAMPLE: Forming committees

Five people are running for three spots on a school committee. How many different committees are possible?

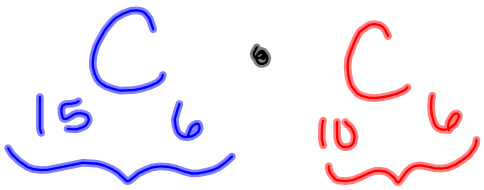
$$_5C_3 = \frac{5!}{2!3!}$$

EXAMPLE: Playing cards

How many different five card poker hands are possible?

$$_{52}C_5 = \frac{52!}{47!\,5!}$$

15 boys and 10 girls are applying to be in Ms. Robson and Ms. John's "Math is Awesome" club. If they can only take 6 guys and 6 girls, how many different ways can they select the club members?



In how many different ways can a 10 question MC exam be answered if each question has 5 answers?

5 5 5 5 5 5 5 5 5 5

5^{10}

You are playing Risk, and rolling 5 dice at the same time. How many possible outcomes are there?

6 6 6 6 6

$6^5 =$

How many different distinguishable permutations are there of the word onamonapia?

$$\frac{10!}{2!\,2!\,3!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2}{2\cdot 2\cdot 3\cdot 2}$$

You are playing a game that requires you to draw two cards from a deck, roll one dice, and flip two coins. How many outcomes are there?

$${}_{52}C_2 \cdot 6 \cdot 2 \cdot 2$$

You are trying out for the first ever PSH Co-ed cricket team. Thirty other students are also trying out, and the roster has a limit of 16 players. How many different teams are possible?

$$_{30}C_{16} = \frac{30!}{14!16!}$$

Homework 8.6

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